List-Heap

Decr key / Add / Merge / Min $O(1)$

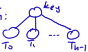
ExtMin $O(n)$

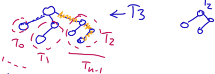
Def. Binomial Tree

$T_0 =$ 

T_1 : 

T_2 : 

T_n : 



y.l: $|T_n| = 2^n$

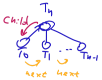
$$|T_n| = 1 + |T_0| + |T_1| + \dots + |T_{n-1}| = 2^n$$

$$2^0 \quad 2^1 \quad \dots \quad 2^{n-1}$$

y.l $depth(T_n) = n$ $\qquad depth(N) = O(\log n)$

y.l $T_n = T_{n-1} + T_{n-1}$

```
Struct Node {
    int key;
    int rank;
(*) Node* parent;
    Node* child;
    Node* next;
    Node* prev;
}
```

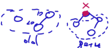


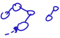
Binomial Heap: $\{$List of T_i $\}_{Min}$

$n = \sum 2^k : \; |T_i| = 2^i$

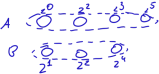
$6 = 2^1 + 2^2$

корней $\leq \log_2 n$





- Add ← Merge — $O(\log n)$
- ⊙ Merge — $O(\log n)$
- Min : ~~$O(\log n)$~~ — $O(1)$
- ExtMin ← Merge — $O(\log n)$
- DecrKey ← siftUp — $O(\log n)$
- Delete ← DecrKey — $O(\log n)$



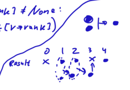
$T_{n+1} \leftarrow T_n + T_n$

Normalize(List of Roots: lst) → Binom. Heap

Result = [None, ...] ≈ log

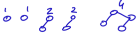
```
for v in lst:
    while Result[v→rank] ≠ None:
        v = Join(v, Result[v→rank])
    Result[v→rank] = v
return Result
```

$O(|lst| + \log n)$



Relaxed Binomial Heap: {List<Node*> Node* min;}

$6 = 2^1 + 2^2$



Merge: $O(1)$

Add: $O(1)$

Min: $O(1)$

DecrKey: ~~$O(\log n)$~~ ✓ $O(1)$, avg

Delete (avg) $O(\log n)$

ExtMin (avg) $O(\log n)$

ExtMin: $|lst| + \log$

Normalize: $|lst| \to \log$

$\varphi = |lst|$ $\Delta\varphi = -|lst| + \log$

$t = |lst| + \log$ $a = t + \Delta\varphi = O(\log)$

BuildHeap: $O(n)$

Fibonacci Heap [1984, Fredman, Tarjan]

Decrkey: siftUp.
O(1)

v {
key,
deg,
marked,
};

Dec cut(v):
v→parent→deg -=1
"удалить v из двусв. списка детей v→parent"
lst.push_back(v); v→marked = false;
// v→parent→marked = 1;
if (v→parent→parent == NULL) return;
if (v→parent→marked) cut(v→parent);
v→parent→marked = 1;

Decrkey(v)
if v→parent == NULL
or not bad(v→parent→key, v→key)
return
cut(v)


v


n

Def Rank(Root) = deg Root

Rank(V) =
= deg(V)
+ V → marked

$\varphi = |lst| + 2\,Marked$

$t = \underline{k} + 1$

$\Delta |lst| = k + 1$

$\Delta\, Marked = -K$

$\Rightarrow a = O(1)$

S_h = min размер поддерева ростом h.

$S_h = 1 + S_0 + \dots + S_{h-2}$

$S_{h-1} = 1 + S_0 + \dots S_{h-1-2}$

$S_0 = 1$

$S_1 = 1$

$$S_h = \underbrace{1 + S_0 + S_1 + \dots + S_{h-3}}_{S_{h-1}} + S_{h-2}$$

$S_h = S_{h-1} + S_{h-2}$

Фиб.

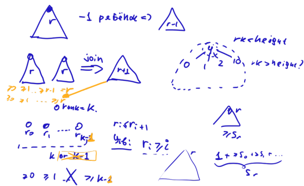
$f_0 = 1$

$f_1 = 1$

$f_n = f_{n-1} + f_{n-2}$

$\Theta\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$

$5 \log_{\frac{1+\sqrt{5}}{2}} N$



r
-1 ребёнок =>
r-1
join
r+1
rk<height
rk>height?
0 1 2 10
r
≥S_r
r_i ≤ r_{i+1}
Утв: r_i ≥ i
1 + ≥S_0 + ≥S_1 + ...
S_r
k or k-1
≥0 ≥1 ≥k-1