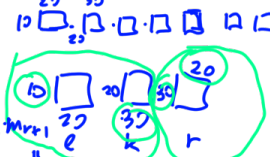
Динамика по подотрезкам.



m:

$m_l \cdot m_{l+1} \cdot \ldots \cdot m_r$

$dp[l][r]$

$A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$

$dp[l][r]$ = минимальное a-ий, чтобы получить произведение $a_l \cdot \ldots \cdot a_r$

$dp[0][n-1] \leq Ans$

$dp[i][i] = 0$

$$dp[l][r] = \min_{l \leq k < r} dp[l][k] + dp[k][r] + T(l,k,r)$$

1) for len=2...n
for l=0..n-len ✓
r= l+len-1
dp[l][r] = ... dp[l+1][r-1]

2) 

for (e=n-1..0)
for (r=e..n-1)
dp[e][r] = ... ✓

Комбинаторные Задачи.

1 2 3	0
1 3 2	1
2 1 3	2
2 3 1	3
3 1 2	4
3 2 1	5

Перестановка $\Rightarrow$ Номер

Номер $\Rightarrow$ Перестановку

Перест $\Rightarrow$ След перест.

Перест $\Rightarrow$ Пред перест

Номер - ?

= # число объектов меньше его

21543

11 [4!]

2 · =/

21 3 [2!] ⇒ 4! + 2! · 2 + 1! = 29

21 4 [2!]

21 53 [1!]

```
used = [false ... false]
ans = 0
for pos = 1..n:          // [1; pos-1]   pos   [pos+1; ...]
    count = 0
    for val = 1..a[pos]-1:
        if !used[val]:
            ++count
    ans += count · factor[n-pos]
```

Номер => переставили.

2 1 . . . 1, 3, 4, 5

~~#2~~ ~~#5~~ #1

1 | 4! = 24 .

2 | 4! = 24 ←

3 | 4! = 24 .

4

5

2 1 | 3! ←

2 3 3!

2 4 3!

2 5 3!

2 1 3 (2!) .

2 1 4 (2!) . 4

2 1 5 (2!) ←

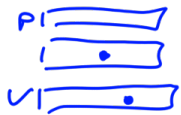
2 1 5 3 (1!)

2 1 5 4 (1!) 2 1 5 4 3

```
used = [false ... false]
id
for pos = 1..n
    for val = 1..n:
        if used[val]:
            continue
        if id >= factorial[n-pos]
            id -= factorial[n-pos]
        else:
            ans[pos] = val
            used[val] = 1
```

Перестановки ⇒ След. перест.

2 1 5 4 3 0



2 1 5 4 ?
2 1 5 ? ?
2 1 ? ? ?
2 ? ? ? ?
5

1, 4, 5, 0

2 3 0 1 4 5

Перест => Пред перест.

23 0 1 4 5

2 3 0 1 4 ?

2 3 0 1 ? ?

2 3 0 ? ? ?

2 3 ? ? ? ?

2 ? ? ? ? ?

2 1 5 4 3 0

Скобочки.

0 1 2 1 2 1 0 1 0
< < < > > < > > < >

< < < > > > #0
< < > < > >
< < > > < >
< > < < > > #3
< > < > < > #0

< >

" < " < " > "

Balance в конце = 0
Balance в time ≥ 0

$O(N^2)$

count[len][bal] =

число способов закончить до конца длины 2N

count[2n][0] = 1

count[2n][≥1] = 0

count[len][bal] =

= count[len+1][bal+1]
+ count[len+1][bal-1]

номер => ncn.

#27.

len

bal

if(id > count[len+1][bal+1])
id -= count[len+1][bal+1]
Ans[len] = ")"

ncn => cncg ncn.

((( )) ( ))
((( )) (? ?
((( )) ? ? ?
((?) ? ? ? ?
((? ? ? ? ?

((( ) (( )))
((,),),
)))

Множество элементов == число

$\{0, 4, 5\} == 2^0 + 2^4 + 2^5 = 49$

$A \cup B$: $A | B$

$A \cap B$: $A \& B$

$A \triangle B$: $A \wedge B$

$A \cup \{x\}$: $A | (1 << x)$
$// A \wedge (1 << x)$
$// A + (1 << x)$

$A \setminus \{x\}$: $// A - (1 << x)$
$// A \wedge (1 << x)$
$A \& \sim (1 << x)$

$A \setminus B$ $A \& \sim B$

$x \stackrel{?}{\in} A$ $(A \& (1 << x)) \neq 0$
$(A >> x) \& 1$

$A \neq \emptyset$ $A \neq 0$

$A \subseteq B$ $(A | B) == B$

$dp[MASK] = |MASK|$

$dp[MASK] = \sum_{v \in MASK} w[v]$

$\{0, \ldots, n-1\}$

$0 \ldots 2^n - 1$

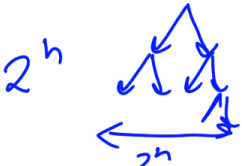
$dp[0] = 0$

for $MASK = 1 \ldots 2^n - 1$: $O(2^n \cdot n)$

  for $v = 0 \ldots n-1$:

    if $v \in MASK$:

      $dp[MASK] += w[v]$




```
go(pos, cur, curw):
    if pos==n:
        dp[cur]=curw
        return
    go(pos+1, cur, curw)
    go(pos+1, cur|(1<<pos),
              curw+w[pos])
```

last = 0
dp[0] = 0
for mask = 1 .. $2^n - 1$:
    if mask == (1 << (last+1)):
        ++last
    dp[mask] = dp[mask \ {last}] + w[last]

last

1 0 1 1 0

dp[0] = 0
for m = 1 .. $2^n - 1$:
    dp[m] = dp[m/2] + (m % 2);

NP-полный

dp[MASK][v] =
= можно ли построить Гам. путь по MASK, зак. v

for v=0..n-1:
  dp[1<<v][v] = 1

for $v = 0..h-1$

$dp'[1 << v] = (1 << v)$

for $m = 1..2^h - 1$

for $v = 0..h-1$:

if $dp[m] \ni v$

for $u = 0..h-1$:

if $u \notin m$ and $go[u][v] == 1$:

$dp'[m \mid (1 << u)] \mid= (1 << u)$

$2^h \cdot h^2$ time

2^h space

$dp'[m] = \{ u \mid dp[m][u] = 1 \}$

$2^h h$ for $m = 1 .. 2^h - 1$: $(dp'[m] \& G0[u]) \neq 0$

for $u = 0 .. h-1$:

20 if $u \notin m$ AND $\exists v: (v \in dp'[m]$ AND $G0[v][u] == 1)$

$dp'[m \mid (1 << u)] = (1 << u)$

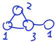
60 

Хроматическое число графа

3^n

2.44^n

2^n



dp[mask] = Мин число цветов в которые можно покр G[mask]

4^n

```
dp[0] = 0
for m = 1..2^n-1:                 S ⊆ m
    for S = 1..2^n-1:             ok[S]  2^n
        if S ⊆ m AND S - нез мн-во:
            dp[m] ← dp[m\S] + 1
dp[(1<<n)-1]
```

Ok[S] = ...

Ok[0] = 0

last = 0

for m = 1..2^h-1:

    if m == (2 << last): // (1 << (last+1))

        last++

    Ok[m] = Ok[m \ {last}] AND (m ∩ Gor[last] == 0);

$\{0 .. n-1\}$

$\langle A, B \rangle$: $A \subseteq B \subseteq \{0 .. n-1\}$

A:	0	0	1	~~1~~	
B:	0	1	1	~~0~~	3^n.

$\binom{n}{0}$ $\binom{n}{1}$ ... $\binom{n}{n}$

2^0 2^1 2^n

$$\sum_{k=0}^{n} \binom{n}{k} 2^k = (2+1)^n$$

$S \le m$

for ($S = m$; $S \neq 0$; $S = prev(S)$)

3^4

3^{4+2^4}

$m = 1012$

1002

0012

0002

$prev(S) = (S-1) \& m$

- $m = 10111011111$
- $S = 10111000000$
- $S-1 = 10111011111$

$m \& (S-1) = 10111011111$